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1990 J. Phys.: Condens. Matter 2 SA443

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J. Phys.: Condens. Matter 2 (1990) SA443–SA448. Printed in the UK

Instabilities in displacement processes in porous media

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Received 31 July 1990, in final form 21 August 1990

Abstract. We review the state of the art in flow instabilities in displacement processes in porous media. Both miscible and immiscible displacements are considered. Effects of mobility ratio, capillary and Peclet numbers are explored. We juxtapose continuum and discrete methods and discuss results and limitations. Emphasis is placed on heterogeneity and its interplay with flow instabilities.

1. Introduction

Flow instabilities in porous media are associated with displacement driven by pressure gradients, where there is (or develops) a spatial contrast in the fluid mobilities, with the mobility decreasing in the flow direction. While there exist certain similarities with the well-studied Saffman–Taylor problem [1, 2] (and perhaps even more with the imperfect Hele–Shaw cell [3]), displacements in porous media are also very different. This is due to the heterogeneity at various scales, which is characteristic of porous media and decisively affects flow patterns [4]. Instabilities in porous media are commonly viewed at scales larger than the pore scale, under the hypothesis that distinct, stable interfaces can be defined within a pore and that the flow is Darcy rather than Stokes. This is taken to be true even in the absence of interfacial tension, for instance for DLA-type processes [5, 6]. It is, therefore, important to point out that shear instabilities of the type discussed in [7, 8] are not considered.

Scale separation, although at a different level, is likewise assumed in the classical treatment of flow instabilities. Here, continuum models relate macroscopic variables in terms of (local) PDEs under the hypothesis that at the pore (or network of pores) scale, concentration or saturation profiles are 'flat'. For primary drainage, appropriate limits on the capillary number (Ca) can be placed [5]. Analogous results for imbibition or for miscible displacement, however, are not known. This continuum approach allows one to consider instabilities as a frontal displacement problem, much like flame propagation [9]. In fact, such frontal instability has been the original definition of viscous fingering [10]. When macroscopic models lose validity (e.g. at larger values of Ca) a non-local, pore level formulation becomes necessary. Several attempts in this direction have recently been made for immiscible displacement [11–15].

Flow instability in porous media is thus intricately related to a few significant variables: the externally imposed ratio in the far-field viscosities and the flow velocity ('non-equilibrium constraint' [16]) and the porous media properties, wettability and pore structure at various scales. Additionally, large-scale effects, such as gravity, the flow geometry, the history of the displacement process and long-range property correlations are of importance. In the following we summarize some of the results obtained on these issues for immiscible and miscible displacements.

2. Continuum models

2.1. Immiscible displacement

Continuum approaches rely on quasi-static hypotheses assuming capillary control at the pore scale. An adequate description of primary drainage only is currently available, imbibition being sensitive to additional effects [17–19]. Continuum approaches are applied in a phenomenological way [20]: a travelling-wave base state, uniform in the direction, y, transverse to the main flow is assumed, which satisfies a parabolic, non-linear PDE. The equation is hypodiffusive for primary drainage (percolation-like), it is a regular diffusion in the case of mobile initial saturations, and it is assumed hyperdiffusive for imbibition. The upstream decay can be exponential or algebraic [21]. These aspects are important for a macroscopically homogeneous medium with small uncorrelated fluctuations, although their effect becomes secondary for strong heterogeneity.

The random microstructure in the above is represented in terms of percolation-dependent properties, which give rise to a transition zone of macroscopic width, scaling with the macroscopic length $\sqrt{k}/Ca$, where k denotes permeability. The validity of the macroscopic predictions is thus limited to large wavelengths, or to weak instabilities. A standard analysis in terms of normal modes ($\mathrm{e}^{\omega t+\mathrm{i}\alpha y}$) yields [21]:

$$\omega = \omega_1|\alpha| + \omega_2\alpha^2 + \ldots \qquad \omega_1 = v\frac{\lambda(1)-\lambda(0)}{\lambda(1)+\lambda(0)}. \tag{1}$$

The leading term ω_1 is due to the contrast in far-field mobility and generalizes the Saffman–Taylor result, predicting large-wavelength instability for $M \equiv \lambda(1)/\lambda(0) > 1$. The appearance of mobility terms $\lambda \sim k/\mu$ in (1) anticipates the importance of permeability heterogeneity. Stabilization occurs at shorter scales due to capillarity and to the existence of a transition zone, both of which contribute a second-order effect, $\omega_2 < 0$ [22]. The short-wavelength limit ($\alpha \gg 1$) can be shown to be stable, in general. However, rigorous results do not exist for the most interesting percolation-like case (hypodiffusive base states).

Other important macroscopic effects include gravity, which leads to a critical flow velocity [23] and flow geometry. Generalizing [24] for radial flow, where algebraic ($t^\sigma \mathrm{e}^{\mathrm{i}n\theta}$) rather than exponential rates apply, immiscible displacement is predicted more stable than in rectilinear flow [25]. A simple transformation links the two geometries at relatively large $Ca, 2(\sigma+1)/Ca = \omega, n/Ca = \alpha$. This suggests the existence of a critical capillary number, $Ca^*(M,\ldots)$, below which the displacement is stable, for all wavelengths, despite $M > 1$.

A weakly non-linear analysis near the onset of instability predicts non-linear stabilization [26]. In an unpublished study, we have also considered the weakly non-linear extension of (1) at small mobility contrast, $\lambda(1) = \lambda(0) + \epsilon$. In rescaled variables, the amplitude equation satisfies

$$A_T + aA_Y^2 + \left(\frac{\omega_1}{\epsilon}\right) H\left\{A_Y\right\} = (-\omega_2)A_{YY} \tag{2}$$

where H is the Hilbert transform. The above is also obtained in the stability of flame fronts [27]. We have found that it predicts stabilization at wavelengths larger (in certain instances twice as large) than the fastest growing wavelength of the linear theory. This hints to finger merging (or pairing), as observed by Homsy [28] in miscible displacements.

Additional features of unstable, immiscible displacement by a continuum formalism have been described in [14]. The asymptotic state of unstable macroscopic fronts (large-scale fingers, tip splitting, etc) is largely unexplored, in the general presumption that features similar to Hele–Shaw flows would emerge. However, a thin-front analysis of the continuum equations in the spirit of [29] shows that the interface conditions are qualitatively different. We hope to report on this issue in the future.

2.2. Miscible displacement

While immiscible displacement is subject to the intricacies of wettability, miscible displacement is, by contrast, easier to describe. This is indeed the case for the dispersion of a passive solute (e.g. see [30, 31]). Surprisingly little is known, however, for the non-linear case, where the fluid viscosity varies with concentration. Contrary to immiscible flows, rigorous pore level results do not exist. Present continuum formulations are therefore only extensions of the passive solute formalism, e.g. with dispersion tensors corresponding to passive solute [15, 32, 33].

With dispersion neglected, continuum models predict a long-wavelength instability identical to (1), and a short-wavelength growth at a finite rate $\omega = \max[\mathrm{d}\ln\mu/\mathrm{d}\xi]$ [23]. By including dispersion and considering sharp, step-like, base states, Tan and Homsy [32] obtained the stabilization cut-off, $\alpha_c = \ln M/2(\epsilon+\sqrt{\epsilon})$ where lengths scale with the dispersivity. The stabilization due to transverse dispersion ($\epsilon = D_\perp/D_\parallel$) is evident. However, one should note the generally small lengths (of the order of pore diameter), below which instability is suppressed. While this is indeed an indicator of the relevance of viscous instabilities in miscible flows (as compared to immiscible, at least for low Ca), the result is also very near the border where continuum models lose validity.

Such possible inadequacies are apparent in [33], where the effects of mechanical dispersion on flow instability were also included. Denoting this effect by $L = a_\parallel q/D_\parallel$, it was shown that for step-like base states the stabilizing cut-off depends on the combination $\eta = (L\ln M/2)\tanh(L\ln M/2) - 1 - \sqrt{\epsilon}$. When $\eta < 0$, which is always satisfied if $L = 0$ (as above) or if the flow rate or the viscosity ratio are small, the cut-off is finite, $\alpha_c \sim (\ln M)\epsilon^{-1/2}/(-2\eta)$. While, when $\eta > 0$, a finite cut-off is not predicted despite the presence of dispersion. The transition requires sufficiently high (but finite) values in the viscosity contrast M, as long as $L \neq 0$. A relevant physical interpretation is currently lacking, and it is possible that this result merely reflects the breakdown of the continuum description under the conditions assumed. In either case, it calls for an improved, non-local description.

It must be borne in mind that in real media, permeability fluctuations and macrodispersion are likely to predominate over pore scale dispersion and to blanket such finer effects. Even under these conditions, however, the effect of viscous instabilities to the macrodispersion needs to be investigated. Recent non-local theories for passive solute dispersion [34, 35] appear promising in this direction. Nevertheless, it must be also pointed out that simulation of the conventional convection–dispersion equations is very common [36–38]. Results obtained bear many similarities with Hele–Shaw displacements (mechanisms of merging and pairing [28] along with shielding, splitting, spreading and stretching [2,36]). Effects of permeability heterogeneity are investigated in [37], where the importance of early events was emphasized.

3. Discrete models

We consider, next, the application of discrete methods, which are best suited for unstable displacement at strong instability. Typical examples are recent 2D simulations without interfacial tension or dispersion, the cut-offs being the pore size [12] or the simulation spacing [11,14,15]. Simulations are performed at a variable mobility ratio. The limit $M = \infty$ corresponds to DLA, the displacing fluid being a fractal cluster. Upon a decrease of M, on the other hand, the displacement pattern consists of a compact core preceded by a fractal interface of dimension varying with M. However, the latter was found to depend on the grid size.

The significance of this sensitivity was recently analyzed by two different approaches [6,15]. Lee *et al* [6] studied the real space renormalization group of a 2D displacement and arrived at the important result that there exist two fixed points only: the Eden point, corresponding to a compact, Euclidean cluster, e.g. as with $M = 1$, which is stable; and the DLA point, corresponding to a fractal cluster, e.g. as with $M = \infty$, which is a saddle point. The conclusion was, therefore, reached that 2D patterns with finite M eventually (sufficiently long times, large length scales or fine enough grids) approach a Euclidean limit (to be understood in the sense of a compact cluster perhaps with a fractal surface). This is valid even in the absence of intrinsic stabilization, such as interfacial tension. Furthermore, they proposed a cross-over scaling law

$$m(R) = R^d F\left(\frac{1}{M}R^{\phi}\right) \tag{3}$$

$$F(x) \sim \begin{cases} 1 & x \ll 1 \\ x^a & x \gg 1 \end{cases} \tag{4}$$

with $d + a\phi = 2$ and $\phi \simeq 0.5$ in 2D. Here R is the radius of gyration in units of lattice spacing, m being the 'mass' of the cluster. Qualitatively similar conclusions were reached by King and Scher [15] in their numerical study. It follows that unstable displacements characterized by transient, non-local, fractal regimes eventually acquire a compact core due to the finite value of M, which allows for the 'base' near injection to be filled [15]. Qualitatively, the early transient can be likened to anomalous dispersion [34,35,39,40]. While the attraction to a Euclidean limit raises hopes for more rigorous continuum formalisms.

The above have consequences in at least two other directions. One is the scale up of numerical or laboratory experiments of highly unstable displacements, which ought to be done with the caution suggested by (3). The other consequence is that heterogeneities of a scale of the order of the cross-over or larger are likely to dominate. Of course, the relevance of large-scale heterogeneities in permeability has been known for a long time by reservoir engineers (see also [41]).

The previous dealt with random porous media, where fluctuations are not correlated and distributions are narrow. Correlated media and wide distributions are expected to significantly influence the displacement. That this is true can be inferred from the anomalous dispersion of a passive solute ($M = 1$) in media correlated at all scales [35, 41, 42], and from the model of Katz and Thompson [43] for a wide conductance distribution, where flow occurs over a percolation cluster. Finite correlations in percolation are expected to only augment finite-size effects. One anticipates the region of capillary control to diminish. In viscous-controlled displacements, the author is aware of two studies [43, 44] where unstable displacement in a correlated permeability field is studied. Furthermore, the use of fractal geostatistics (correlations at all scales) is routinely practiced in modern field-scale simulations [45].

4. Conclusions

In the limited space of this paper, we attempted to highlight some of the issues related to flow instabilities in porous media. We have discussed continuum and discrete approaches, and outlined possible limitations and open problems. For random media, unresolved is the understanding of the imbibition process at various scales, as well as the process of dispersion in unstable flow at low rates (Peclet numbers) and small lengths. For viscous-controlled problems, the recent results of [6, 15] suggest that unstable displacements are asymptotically attracted to non-fractal regimes. A quantitative description of this transition would be of great interest. The development of rigorous continuum models and the examination of large scale effects, such as finger interaction are also needed. Finally, extension of the previous results to 3D is necessary.

For media with correlated fluctuations or wide distributions in properties, fundamental studies similar to [6] are needed. As in passive solute dispersion, finite correlation lengths would augment the size of the region (longer time or length scales) where non-local effects are important. The extension to infinitely long correlations is non-trivial and of great interest. In the context of geologic, porous media, anisotropy also becomes important. These systems are of practical significance (layered porous media). Empirical models for unstable flow [46] have been developed and routinely used. However, their validation awaits further theoretical developments.

Acknowledgments

This paper benefited from many discussions with J-C Bacri, R Lenormand, M Sahimi, and D Salin. The hospitality of the Laboratoire d'Ultrasons, Université Pierre et Marie Curie, where part of this work was completed is also gratefully acknowledged.

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